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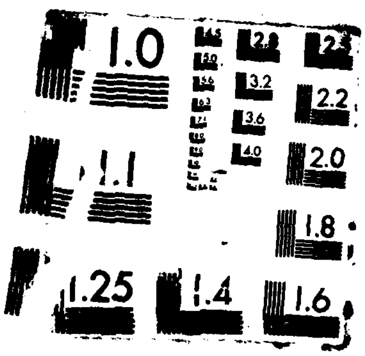
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FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

THE NONLINEAR DYNAMIC RESPONSE OF AN ELASTIC-PLASTIC THIN PLATE UNDER IMPULSIVE LOADING

by

Wang Xintian, Hong Shantao, Weng Zhiyuan

FOREIGN TECHNOLOGY DIVISION

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HUMAN TRANSLATION

FTD-ID(RS)T-0404-87 11 June 1987

MICROFICHE NR: FTD-87-C-000400

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English pages: 28

Source: Guti Lixue Xuebao, Nr. 3, September 1985, pp. 281-295

Country of origin: China
Translated by: FLS, INC.
F33657-85-D-2079
Requester: HQ AFESC/TST
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A-1

The Nonlinear Dynamic Response of an Elastic-Plastic Thin Plate Under Impulsive Loading

by Wang Xintian, Hong Shantao, and Weng Zhiyuan

Tongji University

Paper received on August 25, 1983.

In this paper the effects of the physical and geometrical nonlinearities in a thin plate are treated as equivalent body forces and equivalent loads. Using the concept of influence functions, an analytical method for the thin plate problem with both kinds of nonlinear effects is presented. In theoretical analysis, the influence of plastic region which extends in depth is considered and an incremental formula of plastic strain is derived by applying the incremental plastic theory. In the calculation of practical examples, the numerical solutions for nonlinear dynamic responses of an elastic-plastic thin plate are obtained for various hardening coefficients and different impulsive loads, all of the results are quite regular. (Chinese translations)

I. Preface

Due to the requirement of the antiknock and anti-seismic design of a structure, the nonlinear dynamic response analysis of a structure has been widely recognized.

Two different kinds of nonlinear problems will be encountered in the structure dynamic analysis if the structure undertakes short-duration and large impulsive loads. One is the physical nonlinearity which is the nonlinearity of the relationship between material stress and strain. The other one is geometrical nonlinearity which is the nonlinearity of the relationship between strain and displacement which occurs while the structure deflection exceeds the small deflection range. In the engineering application, more attention is paid to the dynamic response analyses which cover both kinds of nonlinearity. It is a rather complicated problem. Generally, using numerical computation methods is the only way to obtain the approximate solution. Among those numerical methods, the finite element method is the most effective one. The method presented in this paper is an "influence function" numerical method. It is conceived from A. A. Il'yushchin's elastic method for a plastic problem, A. S. Vol'mir's method for a large deflection próblem, and T. N. Lin's method for a dynamic problem. Furthermore, the concept of influence functions is applied to solve the dynamic response of a thin wall problem with both kinds of nonlinear effects. Compared with the finite element method, the method presented in this paper

is simpler. Its computational time is much less than the finite element method. Its precision is higher also.

II. Basic Assumption and the Influence Function of a Simple Supported Plate.

1. Basic Assumption

1) The loading is a suddenly increased uniform load which varies with time.

2) The structure is an elastic-plastic thin plate. Both elastic and plastic regions follow the Kirchhoff assumption.

3) The material of the structure contains the dual-linear hardening characteristic as shown in Fig. 1.

4) The plastic theoretical analysis adopts the isotropic hardening model. The material follows the von Mises yielding function and the associated Prandtl-Reuss flow rule.

5) The inertia force effects at both x and y directions are neglected in the dynamic response calculation.

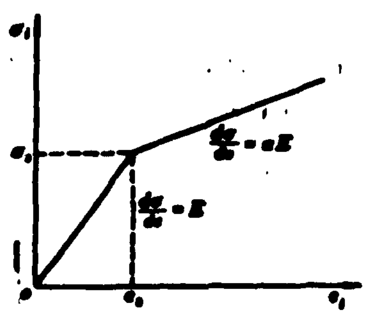


Fig. 1

2. The Influence function of a Simple Supported Plate

The motion differential equation of a thin plate can be written as

$$D\nabla^4 w + \frac{rh}{g}\ddot{w} = q(t) \tag{1}$$

Its resonance frequency is

$$\omega_{mn} = \pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)\sqrt{\frac{Dg}{rh}} \tag{2}$$

Assuming its particular solution is

$$w^* = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} X_m(x) Y_n(y) T_{mn}(t) \tag{3}$$

After normalization, choose $X_m(x) = \sqrt{\frac{2}{a}}\sin\frac{m\pi x}{a}$, $Y_n(y) = \sqrt{\frac{2}{b}}\sin\frac{n\pi y}{b}$ Applying the orthogonality of the trigonometric function, and choosing the initial condition as $w = \dot{w} = 0$ when t=0, we have

$$T_{mn}(t) = \frac{8\sqrt{ab}\,g}{mn\pi^2 rh\omega_{mn}}\int_0^t q(\tau)\sin\omega_{mn}(t-\tau)\,d\tau \tag{4}$$

The dynamic response of a simple supported rectangular plate due to the point load of an unit strength at (ξ, η) is

$$\underline{w}(x, y, t) = \frac{4g}{rh}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\sin\frac{m\pi\xi}{a}\sin\frac{n\pi\eta}{b}}{ab\omega_{mn}^2} \times (1-\cos\omega_{mn}t) \tag{5}$$

This is the Green function of the dynamic response of a simple supported thin plate. After the time and spacial variables in the Equation 5 are separated, the dynamic influence coefficient of a thin plate deflection is

$$G(i,j,k,l,p) = \frac{4g}{rhab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin\frac{m\pi}{a}(i\Delta x)\,\sin\frac{n\pi}{b}(j\Delta y)\,\sin\frac{m\pi}{a}(k\Delta x)\,\sin\frac{n\pi}{b}(l\Delta y)}{\omega_{mn}^2}$$

$$\times\left[1-\cos\omega_{mn}(p\cdot\Delta t)\right] \qquad (6)$$

Equation 6 represents the deflection response at grid (i, j) when time step is equal to p due to the step load of a unit strength act on grid (k, l) when t=0.

Similarly, the static influence coefficient of a thin plate deflection can be obtained from the thin plate static differential equation.

$$S(i,j,k,l) = \frac{4}{\pi^4 abD}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin\frac{m\pi}{a}(i\Delta x)\,\sin\frac{n\pi}{b}(j\Delta y)\,\sin\frac{m\pi}{a}(k\Delta x)}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\,\sin\frac{n\pi}{b}(l\Delta y) \qquad (7)$$

Both dynamic influence coefficient G and static influence coefficient S of a thin plate are called influence function for short in this paper.

III. The Dynamic Responses of an Elastic-Plastic Thin Plate Small Deflection When Hardening

The non-hardening case of the dynamic responses of an ideal elastic-plastic thin plate small deflection has been discussed in Ref [3]. This section will depict the dynamic response of an elastic-plastic thin plate small deflection while considering the hardening.

The motion equation of an elastic-plastic thin plate (small deflection) is written as

$$D\nabla^4 w + \frac{rh}{g}\ddot{w} = q(t) + \bar{q}(x,y,t) \tag{8}$$

where $\bar{q}(x,y,t)$ is the lateral equivalent loading caused by the plastic strain inside the plate, and is written as

$$\bar{q}(x,y,t) = -\frac{E}{1-\nu^2}\left(\frac{\partial^2}{\partial x^2}\int (e_x^p + \nu e_y^p) z dz + \frac{\partial^2}{\partial y^2}\int (e_y^p + \nu e_x^p) z dz + 2(1-\nu)\frac{\partial^2}{\partial x \partial y}\int e_{xy}^p z dz\right). \tag{9}$$

where e_x^p, e_y^p, e_{xy}^p are time varied plastic strain components.

The thin plate is divided into uniform grids while the time history is divided into equal intervals. Considering the case of simple supports of four sides of the plate, substitute the dynamic influence coefficient G shown in Equation 6 into it, then the dynamic response successive substitutional equation of a thin plate at time pΔt is

$$\begin{aligned} w(i,j,p) = & \sum_{k=1}^{M}\sum_{l=1}^{N} G(i,j,k,l,p)[\Delta q(k,l,o) + \Delta\bar{q}(k,l,o)] \\ & + \sum_{r=1}^{p-1}\sum_{k=1}^{M}\sum_{l=1}^{N} \bar{G}(i,j,k,l,p-r)[\Delta q(k,l,r) + \Delta\bar{q}(k,l,r)] \\ & + \sum_{k=1}^{M}\sum_{l=1}^{N}\frac{1}{2}G(i,j,k,l,1)[\Delta q(k,l,p) + \Delta\bar{q}(k,l,p)] \end{aligned} \tag{10}$$

Where $\Delta q(k,l,r)$ is the lateral loading increment act on grid (k, l) at $t=t_r=r\cdot\Delta t$, and $\Delta\bar{q}(k,l,r)$ is the plastic equivalent loading increment act on grid (k, l) at $t=t_r=r\cdot\Delta t$ while hardening is considered. $\bar{G}(i,j,k,l,p-r) = \frac{1}{2}[G(i,j,k,l,p-r) + G(i,j,k,l,p-r+1)]$.

Considering the dual-linear situation of the material stress-strain relations (see Fig. 1), α is a hardening coefficient, σ_s is a plastic yielding limit. Using the Prandtl-Reuss flow rule and the broad sense of the Hooke law, the relationship between pre-plasticized strain increments $\Delta e'_{xp}$, $\Delta e'_{yp}$, $\Delta e'_{xyp}$ and full strain increments Δe_{xp}, Δe_{yp}, Δe_{xyp} at time pΔt is

$$\begin{cases} \Delta e'_{xp} = \frac{A}{T}[(A+\nu B)\Delta e_{xp} + (B+\nu A)\Delta e_{yp} + (1-\nu)C\Delta e_{xyp}] \\ \Delta e'_{yp} = \frac{B}{T}[(A+\nu B)\Delta e_{xp} + (B+\nu A)\Delta e_{yp} + (1-\nu)C\Delta e_{xyp}] \\ \Delta e'_{xyp} = \frac{C}{2T}[(A+\nu B)\Delta e_{xp} + (B+\nu A)\Delta e_{yp} + (1-\nu)C\Delta e_{xyp}] \end{cases} \tag{11}$$

where

$$\begin{cases} A = \frac{1}{2}(2\sigma_{xp-1} - \sigma_{yp-1}) \qquad B = \frac{1}{2}(2\sigma_{yp-1} - \sigma_{xp-1}) \\ C = 3\tau_{xyp-1} \\ T = (1-\nu^2)\sigma^2_{ip-1}\frac{\alpha}{1-\alpha} + A(A+\nu B) + B\,(B+\nu A) + \frac{1-\nu^2}{2}C^2 \end{cases} \tag{12}$$

the stress strength σ_{ip-1} is

$$\sigma_{ip-1} = (\sigma^2_{xp-1} + \sigma^2_{yp-1} + \sigma_{xp-1}\sigma_{yp-1} + 3\tau^2_{xyp-1})^{\frac{1}{2}} \tag{13}$$

The increment form of the plastic strain equivalent loading $\bar{q}$ is

$$\Delta\bar{q} = -\frac{E}{1-\nu^2}\Big(\frac{\partial^2}{\partial x^2}\int(\Delta e'_x + \nu\Delta e'_y)z\,dz + \frac{\partial^2}{\partial y^2}\int(\Delta e'_y + \nu\Delta e'_x)z\,dz$$
$$+ 2(1-\nu)\frac{\partial^2}{\partial x\partial y}\int \Delta e'_{xy}\, z\,dz\Big) \tag{14}$$

Notice that the plastic strain increment formula, Equation 11, derived in this paper which considers the hardening situation, is consistent with the matrix forms used in the elastic-plastic finite element methods which are presented in Ref [16], [18].

The form of the dividing point $\xi(x,y,t)$ between elastic and plastic along the depth in the plate can be expressed as

$$\xi(x,y,t)=\pm\sigma_s\left\{\frac{E}{1-\nu^2}\left[\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)^2+\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)^2-\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)\right.\right.$$

$$\left.\left.\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)+3(1-\nu)\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]^{\frac{1}{2}}\right\}^{-1} \qquad (15)$$

During the successive substitution process, based on the Kirchhoff hypothesis, the full strain increment can be obtained by using the formulas below

$$\begin{cases}\Delta e_{xp}=-z\dfrac{\partial^2(\Delta w_p)}{\partial x^2}\\ \Delta e_{yp}=-z\dfrac{\partial^2(\Delta w_p)}{\partial y^2}\\ \Delta e_{xyp}=-z\dfrac{\partial^2(\Delta w_p)}{\partial x\partial y}\end{cases} \qquad (16)$$

where $\Delta w_p=w_p-w_{p-1}$. The total stress at time $p\Delta t$ is

$$\begin{cases}\sigma_{xp}=\sigma_{xp-1}+\Delta\sigma_{xp}\\ \sigma_{yp}=\sigma_{yp-1}+\Delta\sigma_{yp}\\ \tau_{xyp}=\tau_{xyp-1}+\Delta\tau_{xyp}\end{cases} \qquad (17)$$

For the convenience of comparison, the same rectangular plate used in Ref [3] is selected to calculate the dynamic response of an elastic-plastic thin plate small deflection when the hardening occurs.

The thin plate is shown in Fig. 2, where a=25.4cm, h=1.27cm, $E=7.03\times10^{5}$ kg/cm^{2}, $\nu=0.3$, $r=2.77$ g/cm^{3}, $\sigma_s=2109 kg/cm^{2}$, and its four sides are simple supported. $q=21kg/cm^{2}$, the time interval is chosen as 1/48 of the basic period T_1, $\Delta t=0.223\times10^{-4}$ s. Take the advantage of symmetry, divide the one quarter of the plate into 4x4 grids, and divide five intervals in the depth of the plate. Based on four different conditions $\alpha = 0$, 0.2, 0.5, and 1.0, the results of calculation are plotted in Fig. 3. From Fig. 3, it shows that the maximum deflection decreases while α increases, and the thin plate appeared to have a hardening tendency. The curves are quite regular.

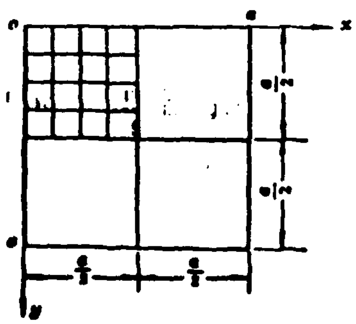


Fig. 2

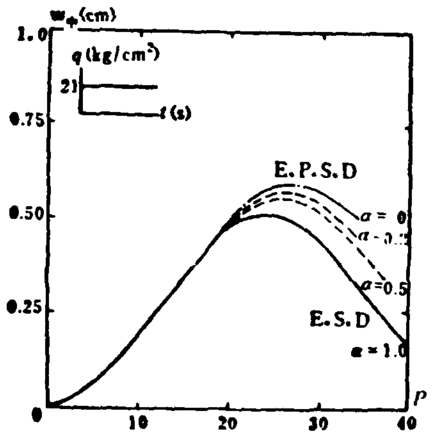


Fig. 3

When the load is further increased, the deflection will exceed the small deflection range. In this case, the geometrical nonlinearity effects must be considered at the same time. This will be discussed in the next section.

IV. Analysis of a Problem which Contains both kinds of Nonlinearities

In the anti-knock and anti-seismic design of a structure, the impulsive loads are usually violent. This often introduces both physical and geometrical nonlinear effects in the structure. Therefore, this sort of problem has obvious engineering significance.

In order to systematically analyze the problem, we shall attack the topic start from the static part then extend to the dynamic part.

1. The static analysis of an elastic-plastic thin plate large deflection

The coordinate system is shown in Fig. 4. Based on the Kirchhoff hypothesis, we have

$$e_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2} = e_{x0} - z\frac{\partial^2 w}{\partial x^2}$$

$$e_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - z\frac{\partial^2 w}{\partial y^2} = e_{y0} - z\frac{\partial^2 w}{\partial y^2}$$

$$e_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial w}{\partial x}\right) - z\frac{\partial^2 w}{\partial x \partial y} = e_{xy0} - z\frac{\partial^2 w}{\partial x \partial y} \quad (18)$$

where e_{x0}, e_{y0}, e_{xy0} are plane strains. The total strain is composed of the elastic strain and the plastic strain. For the plane stress problem, the stress-strain relation is

$$\begin{cases} \sigma_x = \dfrac{E}{1-\nu^2}[(e_x - e_x^p) + \nu(e_y - e_y^p)] \\ \sigma_y = \dfrac{E}{1-\nu^2}[(e_y - e_y^p) + \nu(e_x - e_x^p)] \\ \tau_{xy} = \dfrac{E}{1+\nu}(e_{xy} - e_{xy}^p) \end{cases} \quad (19)$$

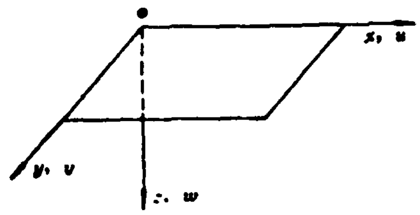


Fig. 4

Substitute Equation 18 into Equation 19, and then integrate it along the entire plate depth, and we have

$$\begin{cases} N_x = \dfrac{Eh}{1-\nu^2}\left[\dfrac{\partial u}{\partial x} + \nu\dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 + \dfrac{\nu}{2}\left(\dfrac{\partial w}{\partial y}\right)^2\right] - \dfrac{E}{1-\nu^2} \\ \qquad \int (e_x^p + \nu e_y^p)\,dz = N_{x0} - \overline{N}_x \\ N_y = \dfrac{Eh}{1-\nu^2}\left[\dfrac{\partial v}{\partial y} + \nu\dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 + \dfrac{\nu}{2}\left(\dfrac{\partial w}{\partial x}\right)^2\right] - \dfrac{E}{1-\nu^2} \\ \qquad \int (e_y^p + \nu e_x^p)\,dz = N_{y0} - \overline{N}_y \\ N_{xy} = \dfrac{Eh}{2(1+\nu)}\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y}\right) - \dfrac{E}{1+\nu}\int e_{xy}^p\,dz \\ \qquad = N_{xy0} - \overline{N}_{xy} \end{cases} \quad (20)$$

where N_{x0}, N_{y0}, N_{xy0} are the film forces of the corresponding middle plane strains, while $\overline{N}_x$, $\overline{N}_y$, $\overline{N}_{xy}$ are the film forces of the corresponding plastic strain in the large deflection condition.

$$\begin{cases} M_x = -D\left(\dfrac{\partial^2 w}{\partial x^2} + \nu\dfrac{\partial^2 w}{\partial y^2}\right) - \dfrac{E}{1-\nu^2}\int (e_x^p + \nu e_y^p)\,z\,dz \\ M_y = -D\left(\dfrac{\partial^2 w}{\partial y^2} + \nu\dfrac{\partial^2 w}{\partial x^2}\right) - \dfrac{E}{1-\nu^2}\int (e_y^p + \nu e_x^p)\,z\,dz \\ M_{xy} = D(1-\nu)\dfrac{\partial^2 w}{\partial x \partial y} + \dfrac{E}{1+\nu}\int e_{xy}^p\,z\,dz \end{cases} \quad (21)$$

The static equation of an elastic-plastic large deflection is

$$\begin{cases} D\nabla^4 w = q + \bar{q} + q^l \\ \nabla^4 \phi = \overline{F} + F^l \end{cases} \tag{22}$$

where q is the lateral load, $\bar{q}$ is the plastic strain equivalent load, and q^l is the large deflection lateral equivalent load.

$$q^l = \frac{\partial^2\phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2\phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2\phi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y} \tag{23}$$

$\overline{F}$ is the plastic strain equivalent body force of the stress function, written as

$$\overline{F} = -E\left(\frac{\partial^2}{\partial y^2}\int e_x^p dz + \frac{\partial^2}{\partial x^2}\int e_y^p dz - 2\frac{\partial^2}{\partial x\partial y}\int e_{xy}^p dy\right) \tag{24}$$

F^l is the large deflection equivalent body force of the stress function, written as

$$F^l = Eh\left(\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right) \tag{25}$$

The relationship between the stress function ϕ, the middle plane film force, and the middle plane strain is

$$N_{x0} = \frac{\partial^2\phi}{\partial y^2} \qquad N_{y0} = \frac{\partial^2\phi}{\partial x^2} \qquad N_{xy0} = -\frac{\partial^2\phi}{\partial x\partial y} \tag{26}$$

$$\begin{cases} e_{x0} = \dfrac{1}{Eh}\left(\dfrac{\partial^2\phi}{\partial y^2} - \nu\dfrac{\partial^2\phi}{\partial x^2}\right) \\ e_{y0} = \dfrac{1}{Eh}\left(\dfrac{\partial^2\phi}{\partial x^2} - \nu\dfrac{\partial^2\phi}{\partial y^2}\right) \\ e_{xy0} = -\dfrac{1+\nu}{Eh}\dfrac{\partial^2\phi}{\partial x\partial y} \end{cases} \tag{27}$$

The successive substitution formula can be written as

$$w_r(i,\ j) = \sum_{k=1}^{M} \sum_{l=1}^{N} S(i, j, k, l)\,[\Delta q_r(k, l) + \Delta q_r^{\prime}(k, l) + \Delta \bar{q}_r(k, l)] \qquad (28)$$

Solutions can be obtained by solving the Equation (28) and the second equation of Equation (22) simultaneously. The entire process can be stated briefly as follows: The load q is increased by the same increment Δq, the rth load increment is called the rth load. Let $\Delta q_r^{\prime} = \Delta \bar{q}_r = \Delta \bar{F}_r = \Delta F_r^{\prime} = 0$, substitute it into Equation (28) and obtain w_r^0. This w_r^0 can be used as the initial value of successive substitution. The first approximate value of the plastic strain increment of this stage can then be calculated by considering the total stress created under the final load of previous stage load, and the load increment Δq of this stage. Find $\Delta \bar{F}_r^0$, $\Delta F_r^{\prime 0}$ and substitute them into the second equation of Equation (22), then $\Delta \phi_r^0$ can be obtained by applying the finite difference method, thus $\Delta q_r^{\prime 0}$ and $\Delta \bar{q}_r^0$ can be calculated. Again substitute both values into Equation (28) to obtain the approximate value after the first successive substitution w_r^1.... Repeat this process until the converge criteria $|\Delta w_r^n - \Delta w_r^{n-1}| < \varepsilon$ is met, which means the difference is negligible. Then add one more load increment Δq. Repeat the entire above processes until the final load reaches q. The final solution w(i, j) is the static solution of the elastic-plastic thin wall large deflection under the load q.

This paper takes the square plate shown in Fig. 2 as a practical example and conducts the calculation by treating it as a static problem. The boundary condition is that the four sides of the plate are hinge-supported. This is the simpler case. Four difference conditions are considered in the calculation: Elastic small deflection (E.S.S), Elastic large deflection (E.L.S), Elastic-Plastic small deflection (E.P.S.S), and Elastic-Plastic large deflection (E.P.L.S). Four resulting curves are plotted in Fig. 5.

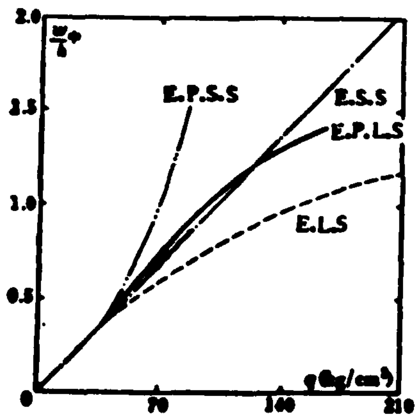


Fig. 5

For the case of elastic large deflection problem of a thin plate, A. C. Vol'mir gave the first stage approximate calculation formula of a square plate,

$$A\zeta^3 + B\zeta = q^*$$

Under the four-side hinge-supported condition, A=7.5, B=22. The nondimensional parameter $\zeta = f_M/h,\ q^* = \frac{qa^4(1-\nu^2)}{Eh^4}$, where f_M is the maximum deflection. The more precise solution should be located

at the left side of this approximate solution, Ref [4]. Meanwhile, based on Ref [10], this paper obtains one curve by using interpolation method. From Fig. 6, it shows the method presented in this paper has higher precision.

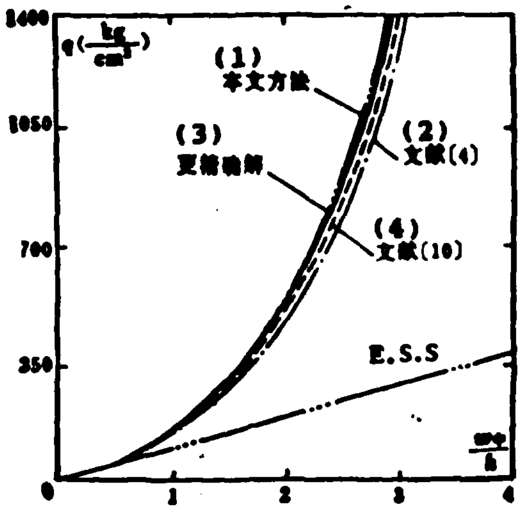


Fig. 6
Key: (1) Method in this paper; (2) Ref [4]; (3) More precise solution; (4) Ref [10]

Fig. 7 and Fig. 8 show the plastic region distribution on the upper, lower surface of the plate and along the depth of the plate of the case which considers large deflection effects. Fig. 9 shows the deflection curves at the different cross sections of the plate which again are the results of the four different cases.

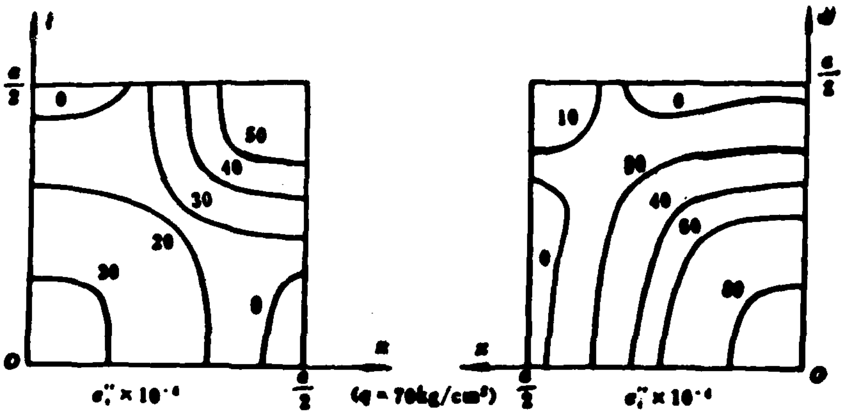
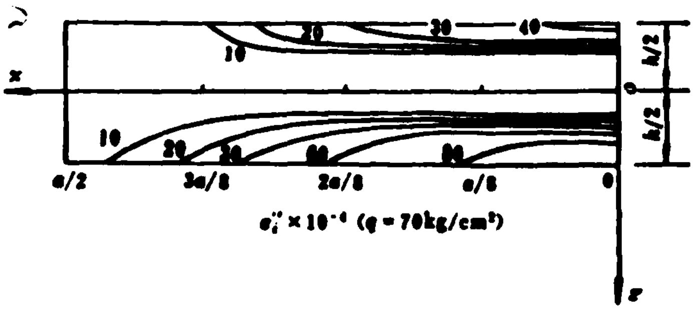
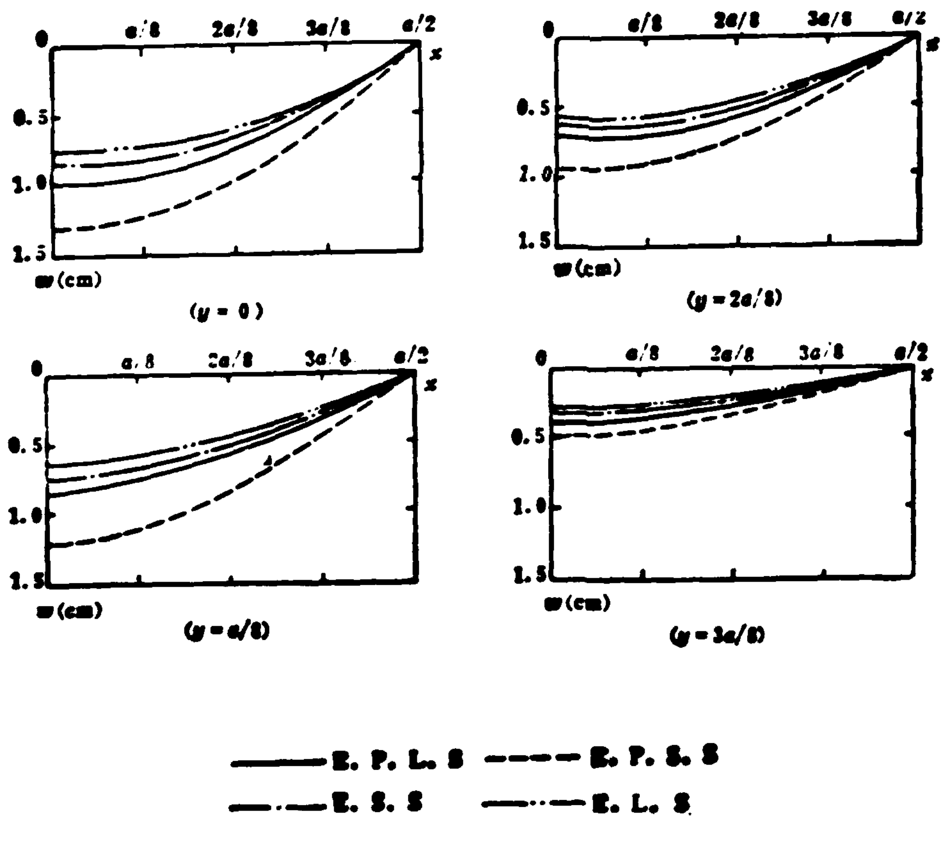


Fig. 7



x
h/2
h/2
0
10
20
30
40
10
20
30
60
80
a/2
3a/8
2a/8
a/8
0
z

Fig. 8



w (cm)
(y = 0)
(y = 2a/8)
(y = a/8)
(y = 3a/8)
E. P. L. S
E. P. S. S
E. S. S
E. L. S

Fig. 9

2. The dynamic response analysis of the elastic-plastic large deflection of a thin plate

The motion equation of the elastic-plastic large deflection of a thin plate can be written as

$$\begin{cases} D\nabla^4 w + \dfrac{rh}{g}\ddot{w} = q + \bar{q} + q^l \\ \nabla^4 \phi = F^l + \bar{F} \end{cases} \tag{31}$$

Compared with the static equation, the above equation contains only one term, an inertia term, more than Equation (22). Other than the time factor need to be added, the expression forms of the stress, strain, internal force, internal torque as well as $\bar{q}$, q^1, $\bar{F}$, F^1, ϕ and etc. for the thin plate in the dynamic case are similar to those in the static case. In this paper, the inertia term is handled by applying T. H. Lin's dynamic problem solving method and introducing the dynamic influence coefficient G(i, j, k, l, p) of a thin plate deflection. The successive substitution formula is

$$\begin{aligned} w(i,j,p) = {} & \sum_{k=1}^{m}\sum_{l=1}^{n} G(i,j,k,l,p)\,[\Delta q(k,l,0) + \Delta\bar{q}(k,l,0) + \Delta q^l(k,l,0)] \\ & + \sum_{r=1}^{p-1}\sum_{k=1}^{m}\sum_{l=1}^{n} \bar{G}(i,j,k,l,p-r)\,[\Delta q(k,l,r) + \Delta\bar{q}(k,l,r) + \Delta q^l(k,l,r)] \\ & + \sum_{k=1}^{m}\sum_{l=1}^{n} \frac{1}{2} G(i,j,k,l,1)\,[\Delta q(k,l,p) + \Delta\bar{q}(k,l,p) + \Delta q^l(k,l,p)] \quad p\Delta t \end{aligned} \tag{32}$$

where $\Delta q^l(k,l,r)$ is the large deflection equivalent load increment which acts on grid (k, l) at time $t = t_r$. The finite difference

incremental form of the second equation of Equation (31) on the grid (i, j) at time $p\Delta t$ is

$$
\begin{aligned}
&20(\Delta\phi_{p,i,j}) - 8(\Delta\phi_{p,i+1,j} + \Delta\phi_{p,i,j+1} + \Delta\phi_{p,i-1,j} + \Delta\phi_{p,i,j-1}) + 2(\Delta\phi_{p,i+1,j-1} \\
&+ \Delta\phi_{p,i+1,j+1} + \Delta\phi_{p,i-1,j+1} + \Delta\phi_{p,i-1,j-1}) + (\Delta\phi_{p,i+2,j} + \Delta\phi_{p,i,j+2} + \Delta\phi_{p,i-2,j} \\
&+ \Delta\phi_{p,i,j-2}) = \Delta F'_{p,i,j} + \Delta\overline{F}_{p,i,j}
\end{aligned}
\qquad (33)
$$

The successive substitution process is similar to the previous case. For the convenience of comparison, the square plate shown in Fig. 2 is still used as a practical example to calculate its dynamic response. The boundary is still the hinge-supported points.

The impulsive load is a suddenly increased platform load. Three difference loads which are q=28, 35, 56 kg/cm^2, respectively, are used in the calculations. Four different cases are analyzed in this paper; they are: elastic small deflection (E.S.D), elastic large deflection (E.L.D), elastic-plastic small deflection (E.P.S.D) and elastic-plastic large deflection (E.P.L.D). The dynamic response curves are then obtained as shown in Fig. 10, 11, and 12. In order to discuss the geometrical nonlinear effect, the impulsive load was increased to q=70 kg/cm^3 in this paper. The cases of E.S.D, E.L.D, and E.P.L.D were calculated. The dynamic response curves for these three cases are shown in Fig. 13.

This paper also calculates the cases when the impulsive load is a triangular and a square function. The associated dynamic response curves are shown in Fig. 14 and 15.

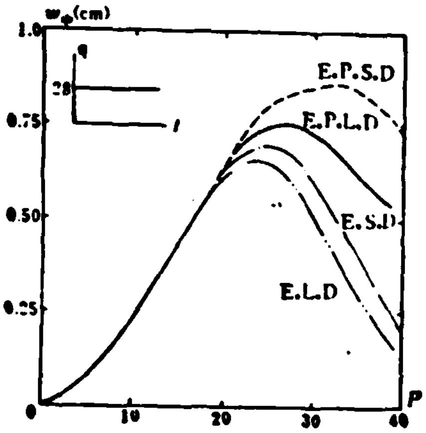


Fig. 10.

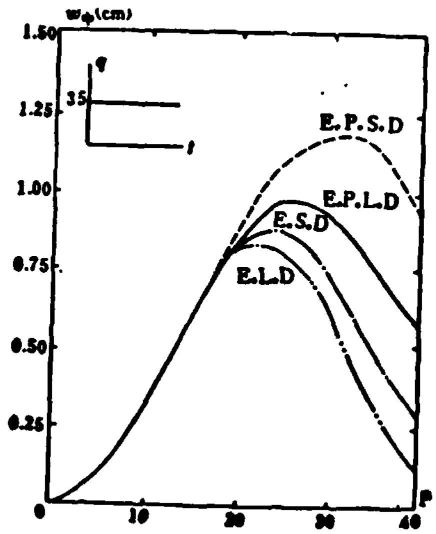


Fig. 11.

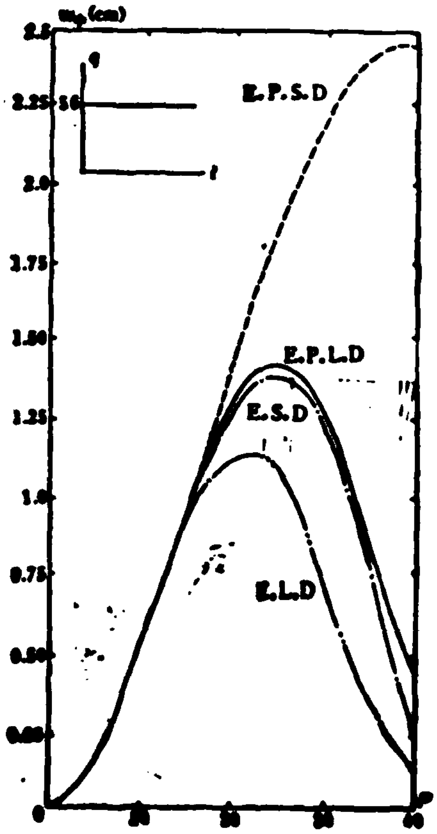


Fig. 12.

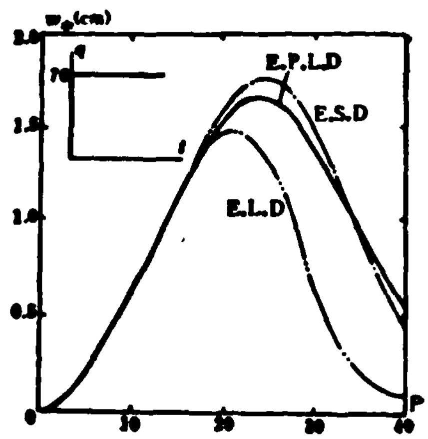


Fig. 13

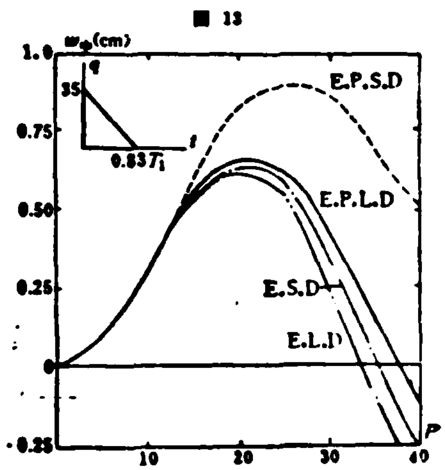


Fig. 14.

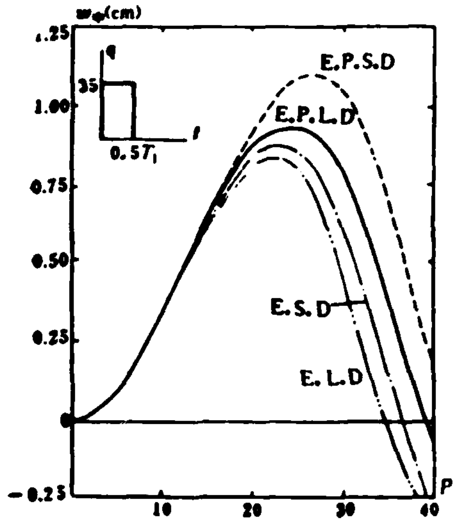


Fig. 15.

From these curves, we notice that the thin plate is "softened" when the plastic effect is considered, while the thin plate is "hardened" when the geometrical nonlinear effect is considered. Furthermore, the geometrical effect abruptly increases along with the increasing of the impulsive load. The curves show the particular regularity.

Due to there being no literature which discusses the influence of plastic region which extends in depth and also considers the thin plate dynamic response with both kinds of nonlinearity effects, a direct comparison and discussion can not be made. However, a "degraded" situation is made in this paper to conduct an indirect comparison and discussion.

Ref [11] applies the finite difference method to the studying of the dynamic response of a thin plate elastic-plastic small

deflection problem. That paper uses nondimension parameters. For the case of the four-side simple supported square plate, we have

$$P_y=\frac{M_0}{0.048a^2} \quad \delta_y=0.00406\frac{P_y a^4}{D} \quad T_1=\frac{2\pi a^2}{18.73}\sqrt{\frac{m}{D}}$$
$$q'=\frac{q}{p_y} \quad \bar{\delta}=\frac{\delta}{q\delta_y} \quad \theta=\frac{t}{T_1} \tag{34}$$

where $M_0=\frac{1}{4}\sigma_s h^2$ is the thin plate yielding moment. Notice that Ref [11] uses the finite element method to obtain T_1 which is inconsistent with the result obtained by the analytical method presented in this paper. The period used in Ref [11] was converted into the analytical solution value when plotted in Fig. 16. For the square plate shown in Fig. 2, $q'=2$ is equivalent to $q=54.8kg/cm^2$. From Fig. 16, the curve obtained by the method in this paper in terms of calculating the case which degraded to the elastic-plastic small deflection, matches quite well with that obtained by using the finite element method presented in Ref [11]. Both curves appear even closer for the maximum deflection.

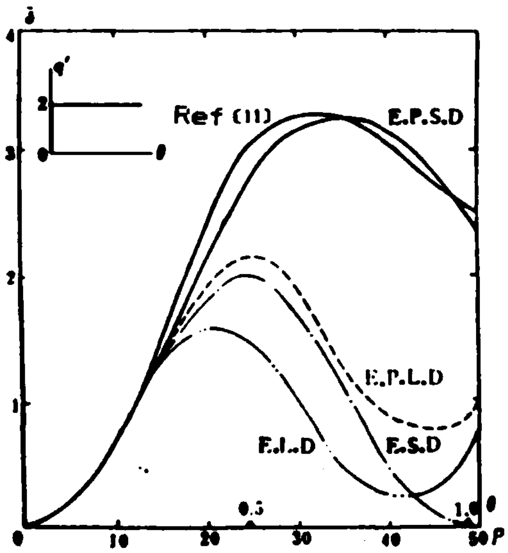


Fig. 16

If the plastic effect is neglected in the case of two nonlinearities, the problem will "degrade" to an elastic large deflection condition. The results of such a "degraded" case by using the method presented in this paper, therefore, can compare with those presented in Ref [6].

Let's introduce the related parameters in Ref [6],

$$\omega^2 = \frac{4\pi^4}{a^4}\frac{Dg}{rh} \qquad \mu^2 = \frac{E\pi^4 g}{8a^4 r} \qquad P = \frac{16g}{\pi r h} q \tag{35}$$

Ref [6] points out that the maximum amplitude A_{max} of the dynamic response of an elastic thin plate large deflection in fact is one of the real roots of Equation $E(A) = A^3 + \frac{2\omega^2}{\mu^2}A - \frac{4p}{\mu^2} = 0$. Obviously, the solution based on the elastic small deflection theory can be obtained if $A^3 = 0$. This paper uses w_{LM}、w_{SM} to represent the maximum deflection of an thin plate elastic dynamic response which is obtained according to the large deflection theory and small deflection theory, respectively. It must be pointed out here that only the first term (N=1) of the trigonometric series is taken in Ref [6] which implies that the thin plate is analyzed by treating it essentially as a single degree of freedom system. Both w_{LM}、w_{SM} are the first stage approximation only. This paper applies the method of influence function where N=7 is taken in the trigonometric series. The results are listed in Table 1.

Table 1

(1)荷 载 ($\frac{kg}{cm^2}$)	w_{SM}(cm)			w_{LM}(cm)		
	(2) 文献〔6〕	(3) 本文方法	(4) 误 差 (%)	(2) 文 献〔6〕	(3) 本文方法	(4) 误差(%)
35	0.923	0.881	4.85	0.845	0.813	5.06
42	1.108	1.056	4.88	1.001	0.953	5.09
56	1.477	1.408	4.89	1.264	1.205	4.87
70	1.846	1.761	4.88	1.494	1.427	4.68

Key: (1) Load; (2) Ref [6]; (3) Method in this paper; (4) Error.

In order to further eliminate the effect of the grid division in the finite difference form and the number of the term in the trigonometric series, this paper introduces a large deflection influence coefficient k_e:

$$k_e = w_{LM}/w_{SM} \tag{36}$$

This parameter reflects the reducing rate of the maximum deflection of a thin plate dynamic response when a large deflection effect is considered. Thus, Table 2 is obtained.

Table 2.

(1)荷 载 ($\frac{kg}{cm^2}$)	(2) 按文献〔6〕求得的 k_e	(3) 按本文方法求得的 k_e	(4) 误 差 (%)
35	0.9252	0.9233	0.21
42	0.9037	0.9019	0.20
56	0.8555	0.8557	0.02
70	0.8091	0.8106	0.18

Key: (1) Load; (2) k_e obtained by Ref [6]; (3): k_e obtained by this paper; (4) Error.

From both Table 1 and 2, the dynamic responses of an elastic large deflection obtained by using the influence method presented in this paper have higher precision.

This paper also calculates the case of choosing N=1 in the trigonometric series, and compares the result with Ref [6]. It is found that the error can be further decreased to 2-3%.

In the practical example calculation of finding the dynamic responses of an elastic-plastic large deflection, when even the time interval Δt is doubled, the difference of the maximum deflection will not exceed 10% in the $q=56 kg/cm^2$ case. This states that the convergence of this paper's method is better also.

In a word, the thin plate dynamic response curves of different conditions obtained by using the method presented in this paper, contain the particular regularity. Because no compatible literature was found, this paper compared the results of the "degraded" condition with the corresponding literature. The results show remarkably well. This explains indirectly that it is feasible to analyze the problem with both kinds of nonlinear effects by using the influence function concept.

V. Discussion

1) This paper applies the concept of influence function, considers the influence of the plastic region which extends in depth and in the x, y direction of the plate, and analyzes the thin plate dynamic response problem with both kinds of nonlinear effects. Through calculation and comparison of the results of a degraded case with the related literature practical examples, it indirectly proves that the method presented in this paper is feasible. From the given elastic solution, this paper solves the influence function. For those more complicated problems, if their elastic solutions are not given, they can be found through finite element methods as pointed out in the Ref [3] abstract. Basically, the method presented in this paper, therefore, can be applied to the nonlinear dynamic response analysis of complicated structures whose elastic solutions are given. The calculation time takes around 35 minutes for each case in a 719 machine.

2) It is noticed through calculation in examples that the physical nolinearity makes a thin plate "soften" while the geometrical nolinearity makes it "harden". Although they both are coupling each other, their effects on the structure are contrary. Along with increases of the impulsive loads, both nonlinear effects build up; however, the effect of geometrical nonlinearity builds up abruptly. From the response curves, we noticed that the majority of a structure E.P.L.D curve fell inside of the E.S.D curve when the impulsive load reached a certain level. N. Jones

pointed out that the plate strain rate is the primary factor when the impulsive loads are small, while the film force of the plate middle plane is the primary one when the impulsive loads are large. However, it lacked precise numerical results. This paper has now given the quantitative results and the associated conclusions. Moreover, they are consistent with the viewpoint of N. Jones.

3) After considering the geometrical nonlinear effect, the neutral plane of a thin plate deviates from the plate middle plane. The plastic strain of the plate extends in depth, its distribution is not symmetrical with respect to the plate middle plane. The examples show that such a situation can cause the reduction of the elastic core in the plate layer, thus forming a plastic yielding region. This phenomenon is very significant while studying the development regularity of a thin plate plastic region or a plastic hinge line along with time.

4) When the impulsive loads are large, under a finite deflection situation, we notice, from the calculated curves, the ratio between the difference of two maximum deflections obtained from a thin plate E.P.S.D curve and E.S.D curve, and the maximum deflection of E.S.D curve, far exceeds the value under the 30% conclusion drawn by some literature which is based on the small deflection theory. When the impulsive load is large, the thin plate dynamic response has exceeded the small deflection region. Therefore, the geometrical nonlinear effect has to be taken into

account; otherwise, the error will abruptly enlarge along with the increase of load. This is also where the significance of this paper lies.

5) The nonlinear dynamic response of an elastic-plastic thin plate studied in this paper belongs to the cases where the ratio of the maximum deflection and the thickness of plate is not too large. Because when the deformation of an elastic-plastic plate is large, the precise analysis becomes complicated, and it is beyond the scope of this paper.

6) The method presented in this paper can only obtain the approximate solutions, because this method adopts the following approximations to find out the numerical solutions. They are a successive substitution method to approaching the solution step by step, a time and spacial variables separation method, finite difference forms as well as the method of finding a plastic strain increment of the current step by using the final full stress of the previous step, and so forth.

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